Sphärische Trigonometrie - Formeln

1. **Rechtwinklige Kugeldreiecke**

$$\sin(\alpha) = \frac{\sin(a)}{\sin(c)} \qquad \sin(\beta) = \frac{\sin(b)}{\sin(c)} \qquad \cos(\alpha) = \frac{\tan(b)}{\tan(c)} \qquad \cos(\beta) = \frac{\tan(a)}{\tan(c)}$$

$$\tan(\alpha) = \frac{\tan(a)}{\sin(b)} \qquad \tan(\beta) = \frac{\tan(b)}{\sin(a)} \qquad \cos(c) = \cos(a) \cdot \cos(b)$$

$$\cos(a) = \frac{\cos(\alpha)}{\sin(\beta)} \qquad \cos(b) = \frac{\cos(\beta)}{\sin(\alpha)} \qquad \cos(c) = \frac{1}{\tan(\alpha) \cdot \tan(\beta)}$$

2. **Beliebige Kugeldreiecke**

   a) **Sinus-Satz**

$$\frac{\sin(a)}{\sin(\alpha)} = \frac{\sin(b)}{\sin(\beta)} = \frac{\sin(c)}{\sin(\gamma)}$$

   b) **Seiten-Cosinus-Satz**

$$\cos(a) = \cos(b) \cdot \cos(c) + \sin(b) \cdot \sin(c) \cdot \cos(\alpha)$$
$$\cos(b) = \cos(a) \cdot \cos(c) + \sin(a) \cdot \sin(c) \cdot \cos(\beta)$$
$$\cos(c) = \cos(a) \cdot \cos(b) + \sin(a) \cdot \sin(b) \cdot \cos(\gamma)$$
$$\cos(\alpha) = \frac{\cos(a) - \cos(b) \cdot \cos(c)}{\sin(b) \cdot \sin(c)}$$
$$\cos(\beta) = \frac{\cos(b) - \cos(a) \cdot \cos(c)}{\sin(a) \cdot \sin(c)}$$
$$\cos(\gamma) = \frac{\cos(c) - \cos(a) \cdot \cos(b)}{\sin(a) \cdot \sin(b)}$$

   c) **Winkel-Cosinus-Satz**

$$\cos(\alpha) = -\cos(\beta) \cdot \cos(\gamma) + \sin(\beta) \cdot \sin(\gamma) \cdot \cos(a)$$
$$\cos(\beta) = -\cos(\alpha) \cdot \cos(\gamma) + \sin(\alpha) \cdot \sin(\gamma) \cdot \cos(b)$$
$$\cos(\gamma) = -\cos(\alpha) \cdot \cos(\beta) + \sin(\alpha) \cdot \sin(\beta) \cdot \cos(c)$$
$$\cos(a) = \frac{\cos(\alpha) + \cos(\beta) \cdot \cos(\gamma)}{\sin(\beta) \cdot \sin(\gamma)}$$
$$\cos(b) = \frac{\cos(\beta) + \cos(\alpha) \cdot \cos(\gamma)}{\sin(\alpha) \cdot \sin(\gamma)}$$
$$\cos(c) = \frac{\cos(\gamma) + \cos(\alpha) \cdot \cos(\beta)}{\sin(\alpha) \cdot \sin(\beta)}$$

3. **Flächen**

Zweiecksfläche $F_2 = \frac{\alpha}{360^\circ} \cdot 4 \cdot \pi \cdot r^2$

Dreiecksfläche $F_\Delta = \pi \cdot r^2 \cdot \frac{\alpha + \beta + \gamma - 180^\circ}{180^\circ}$

Kugelfläche $F = 4 \cdot \pi \cdot r^2$